

Superluminal motion and Lorentzian symmetry breaking and repairing in two-metric theories

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Abstract

The new results by OPERA collaboration claim the discovery of superluminal neutrinos. Superluminal particles have to break Lorentzian symmetry or causality principle. The method discussed gives us the possibility to reintroduce Lorentzian symmetry without breaking of causality.

1 Lorentzian symmetry or causality?

New measurements of high energy neutrino speed by OPERA collaboration [1] suggest the superluminal motion, i.e. motion of particle along space-like world line. Due to theoretical difficulties the discussion of superluminal neutrinos generates the great number of criticism and hypotheses (see [2] and references therein).

If the Lorentzian symmetry still valid, all space-like directions of space-time are equivalent. The possibility of superluminal motion means the possibility of motion backward in time. Actually *it is backward time motion* in some high-speed inertial systems.

So, if one believes in superluminal motion, one has to choose what principle he eliminates: *Lorentzian symmetry or causality*. The causality looks more important. Nevertheless, Lorentzian symmetry is extremely useful tool, which lays in basis of modern physics.

2 “Lorentzian” symmetry of elastic media and its breaking

Let us imagine perfect elastic media with sound waves described by wave equation

$$\left(\frac{\partial^2}{\partial t^2} - u^2 \Delta\right) \mathbf{w} = 0, \quad (1)$$

where u is speed of sound, $0 < u < c$.

Sound waves equation has the sonic “Lorentzian” symmetry with speed of sound instead of speed of light. Sonic “Lorentzian” symmetry preserve the “sonic interval” (“sonic metric”)

$$ds_{sonic}^2 = -u^2 dt^2 + dx^2 + dy^2 + dz^2 = h_{MN} dX^M dX^N.$$

This sonic “Lorentzian” symmetry is broken by non-elastic phenomena (e.g. light). Non-elastic phenomena make possible supersonic motion.

The true Lorentzian symmetry preserve the “true interval” (“true metric”)

$$ds_{true}^2 = -c^2 dt^2 + dx^2 + dy^2 + dz^2 = g_{MN} dX^M dX^N.$$

Existence of the elastic medium breaks the true Lorentzian symmetry of space time. It provides us with preferable coordinate system, the system of medium.

Actually the medium breaks true Lorentzian symmetry at *the level of solutions of field equations*. At *the level of field equations itself* the true Lorentzian symmetry is still valid (“repaired”).

To repair Lorentzian symmetry, let us introduce 4-velocity of elastic medium U^M ($U^M U_M = -1$, $U_M = U^N g_{NM}$). Projectors P_{MN} (to vectors parallel to U^M) and \bar{P}_{MN} (to vectors orthogonal to U^M) are defined

$$P_{MN} = -U_M U_N, \quad \bar{P}_{MN} = g_{MN} - P_{MN}.$$

Sonic metric h_{MN} is defined in terms of true metric g_{MN} and 4-velocity through the projectors

$$h_{MN} = \frac{u^2}{c^2} P_{MN} + \bar{P}_{MN}, \quad h_{MN} h^{NK} = \delta_M^K.$$

The covariant (Lorentzian symmetric) form of equation (1) is

$$\nabla_M (h^{MN} \nabla_N) \mathbf{w} = 0. \tag{2}$$

3 “Elastic medium” of space-time

Similarly to breaking of sonic “Lorentzian” symmetry and preserving of “true” Lorentzian symmetry in the elastic media one could think on breaking Lorentzian symmetry by superluminal neutrinos.

It requires introducing of some “medium”, which generates “light metric” h_{MN} , like “sonic metric” in the example above. This light metric is the metric, which interact with almost all sorts of matter, so it dictates the maximal speed of almost all interactions, i.e. the speed of light c . The light “Lorentzian” symmetry preserves the light metric.

Light Lorentzian symmetry could be broken by some sorts of matter, including neutrinos. Nevertheless, there is also the “true Lorentzian” symmetry, which preserves the “true metric” g_{MN} . The true metric dictates the other maximal speed $c' > c$.

One could rewrite all the standard Lagrangians using light metric for luminal and subluminal matter and true metric for superluminal matter.

The “medium”, which generates light metric plays the role partially similar to historically banned luminiferous aether.

One could easily construct large number of two-metric models starting from different models of relativistic elastic media. The geometrical method of relativistic elastic media modeling is developed in the series of papers [3]–[8].

Different two-metric approaches to interpretation of OPERA results were considered in papers [9], [10], [11], [12], [13].

4 The example

Let us sketch the simplest two-metric model.

To describe “medium” we use one scalar field τ (“luminiferous scalar”) with light-like gradient.

The light metric h_{MN} specified by relation¹

$$h_{MN} = g_{MN} + (\partial_M \tau) (\partial_N \tau). \quad (3)$$

The inverse light metric is

$$h^{MN} = g^{MN} - \frac{(\partial^M \tau)(\partial^N \tau)}{1 + (\partial_K \tau)(\partial^K \tau)}, \quad (4)$$

where

$$g_{MN} g^{NL} = \delta_M^L, \quad h_{MN} h^{NL} = \delta_M^L, \quad \partial^M \tau = g^{MN} \partial_N \tau.$$

The “regular” fields φ are described by standard field theory action $S_{standard}[\varphi, g_{MN}]$ with true metric g_{MN} replaced by light metric h_{MN}

$$S_{regular}[\varphi, g_{MN}, \tau] = S_{standard}[\varphi, g_{MN} \rightarrow h_{MN}].$$

We have the following relation for the variation of light metric

$$\delta h_{MN} = \delta g_{MN} + (\partial_M \delta \tau) (\partial_N \tau) + (\partial_M \tau) (\partial_N \delta \tau).$$

So, all the “regular” fields described by the standard field equations with “true” metric g_{MN} replaced by “light” metric h_{MN} :

$$\left. \frac{\delta S_{regular}}{\delta \varphi} \right|_{g_{MN} \rightarrow h_{MN}} = \left. \frac{\delta S_{standard}}{\delta \varphi} \right|_{g_{MN} \rightarrow h_{MN}}.$$

The gravitational field also could be considered as “regular” field, *if there is no non-regular fields*

$$\left. \frac{\delta S_{regular}}{\delta g_{MN}} \right|_{g_{MN} \rightarrow h_{MN}} = \left. \frac{\delta S_{standard}}{\delta g_{MN}} \right|_{g_{MN} \rightarrow h_{MN}}.$$

The standard action obeys the Lorentzian symmetry, so all the regular fields obey “light” Lorentzian symmetry.

To break the light Lorentzian symmetry one could introduce some “non-regular” fields ϕ , which interacts with true metric g_{MN} .

$$S[\varphi, g_{MN}, \tau, \phi] = S_{regular}[\varphi, g_{MN}, \tau] + S_{non-regular}[g_{MN}, \tau, \phi] \quad (5)$$

Field equation for the luminiferous scalar τ has the form

$$\frac{\delta S}{\delta \tau} = \partial_M \left(2(\partial_N \tau) \left. \frac{\delta S_{standard}}{\delta g_{MN}} \right|_{g_{MN} \rightarrow h_{MN}} \right) + \frac{\delta S_{non-regular}}{\delta \tau}.$$

Due to Einstein equations

$$\left. \frac{\delta S_{standard}}{\delta g_{MN}} \right|_{g_{MN} \rightarrow h_{MN}} = \frac{\delta S_{regular}}{\delta g_{MN}} = - \frac{\delta S_{non-regular}}{\delta g_{MN}}.$$

If we introduce no action for non-regular fields, then “true” metric and “luminiferous scalar” are non-observable, and “light metric” is the only physical metric.

¹E.g. if $g_{MN} = \text{diag}(-c'^2, 1, 1, 1)$, $\tau = v \cdot t$, $v < c'$,
then $h_{MN} = \text{diag}(-c'^2 + v^2, 1, 1, 1) = \text{diag}(-c^2, 1, 1, 1)$, $c = \sqrt{c'^2 - v^2}$.

5 Toy model

The simplest two-metric toy model involve one “regular” scalar field φ and the only “non-regular” scalar ϕ , which is small perturbation to luminiferous scalar

$$\tau = \tau_0 + \phi = v \cdot t + \phi. \quad (6)$$

True Lorentzian metric $g_{MN} = \text{diag}(-c'^2, 1, 1, 1)$ is fixed. Let $(\partial\varphi)_g^2 = g^{MN}(\partial_M\varphi)(\partial_N\varphi)$, $(\partial\varphi, \partial\tau)_g = g^{MN}(\partial_M\varphi)(\partial_N\tau)$. The action (5) is combination of two scalars with two metrics (3),(4)

$$S[\varphi, \tau] = \int d^4x [-(\partial\varphi)_h^2 - (\partial\tau)_g^2] = \int d^4x \left[-(\partial\varphi)_g^2 + \frac{(\partial\varphi, \partial\tau)_g^2}{1 + (\partial\tau)_g^2} - (\partial\tau)_g^2 \right]. \quad (7)$$

The velocities of small perturbations of τ (or ϕ) and φ are c' and $c = \sqrt{c'^2 - v^2} < c'$.

The toy model is probably the simplest one. It coincides with the more complex massive vector model presented in paper [13] in the case of gradient field, i.e. if $\psi_M = \partial_M\tau$.

6 Conclusions

The paper demonstrates the method of theory construction, which could be used for relativistic elastic media models or theories with superluminal particles. The method admits correspondence with standard relativistic theories, it preserves Lorentzian symmetry and causality. The appropriate limit reproduces the standard field theories with no superluminal motion.

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